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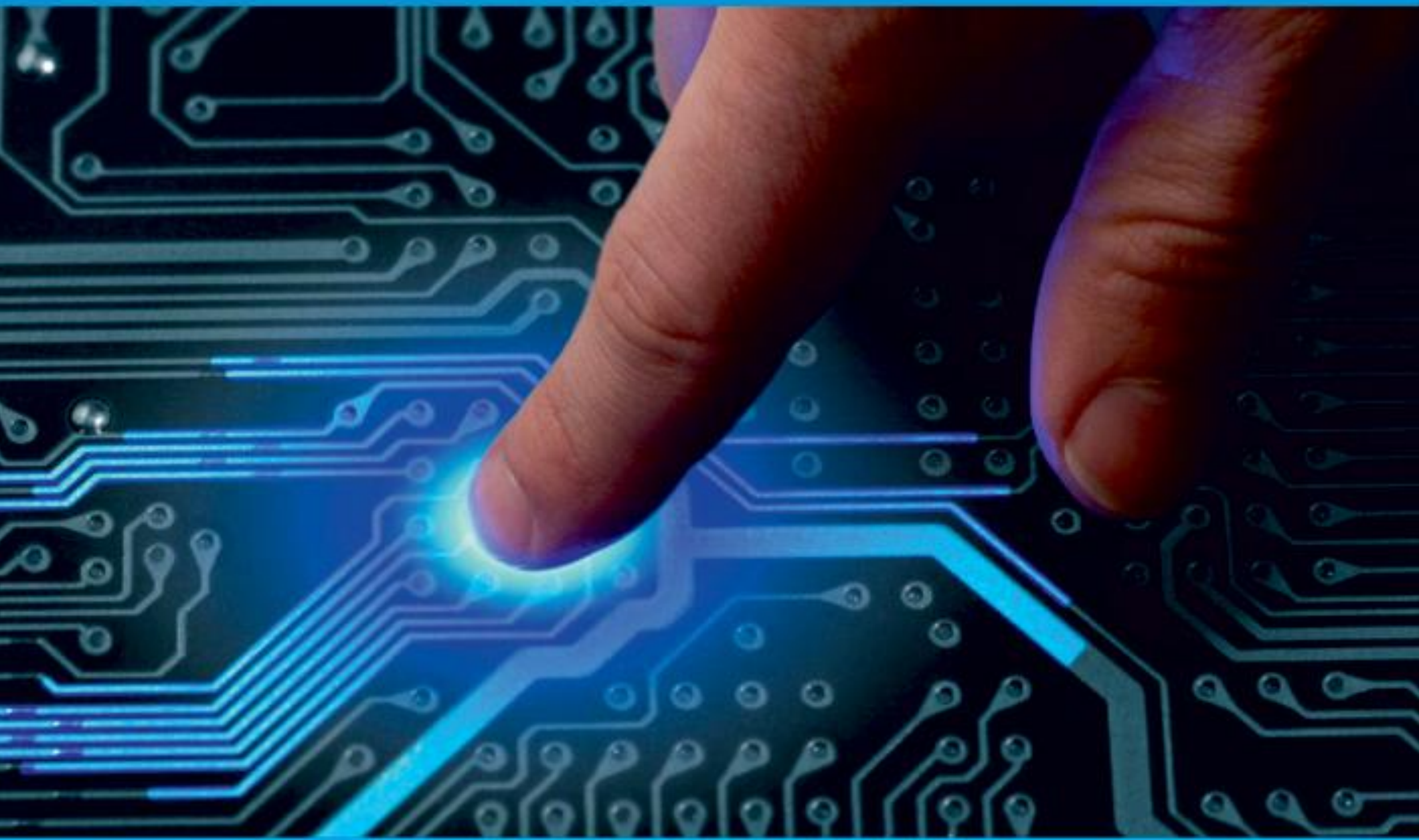

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Hinge Total Domination in Binary Operators Specifically Join of Graphs

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ABSTRACT: The intend of the paper is to grant the centrality of the hinge total domination in connected graphs. In particular, the hinge total domination in join of graphs with some theorems and examples by using resolving sets and locating sets.

KEYWORDS: Resolving hinge total dominating set, locating hinge total dominating set, join, neighborhood, distance.

I. INTRODUCTION

Domination in graphs has been extensively researched branch of graph theory. The rigorous study of dominating set in graph theory began around 1960, even though the subject has historical roots dating back to 1862 when De Jaenisch studies the problem of determiningthe minimum number of queens which is necessary to cover an nxn Chessboard. In 1958, Berge defined the concept of domination number of a graph. In 1962, Ore[1] and in 1977 Cockayne[2] gave extensive survey of results know at that time about dominating set in graph.

Total domination in graphs was introduced by Cockayne, Dawes and Hedetniemi[3] in 1980. The literature on this topic has been detailed explained in the two excellent domination books by Haynes, Hedetniemi, and Slater. In 2009, the more interesting concepts in total domination in graphs has arisen from a computer program Graffiti.pc[4] that has generated several hundred conjectures on total domination.

On one hand, hinge domination in graphs is a new parameter for domination that was accepted and published in 2018. It was introduced by Kavitha B.N. and Indrani Kelkar[7].
Further in 2022, Consistente and Cabahug[8] combined the concepts of the hinge domination and total domination of graphs to form a new variation called hinge total domination of all graphs under the consideration of nontrivial, simple, undirected and finite graphs.

In 2023, Monsanto, Rara[10] published a paper on resolving dominations in graphs under some Binary Operations which gave idea of domination in join of two graphs, corona graphs and lexicographic product graphs and is still open for many possible studies.

Moreover, this paper investigates how the hinge total dominating set behaves in join of graphs, specifically on connected graphs.

II. PRELIMINARIES

Definition 2.1

A set S is called a **total dominating set** of G if for every vertex in V, including those in S is adjacent to at least one vertex in S. The cardinality of a minimum total dominating set in G is called the total domination number of G and denoted $\gamma_t(G)$. Also, a dominating set S of a graph G is a total dominating set if the induced subgraph <S> has no isolated vertices.

Definition 2.2

A set S of vertices in a graph G = (V(G), E(G)) is a **hinge dominating set** if every vertex u ∈ V\S is adjacent to some vertex v ∈ S and a vertex w ∈ V\S such that (v,w) is not an edge in E(G). The hinge domination number $\gamma_h(G)$ is the minimum size of a hinged dominating set.

Definition 2.3

A **hinge total dominating set** of a graph G is a set S of vertices of G such that S is both a hinge dominating set and total dominating set. The hinge total domination number $\gamma_{ht}(G)$, is the minimum cardinality of a hinge total dominating set of G.

Definition 2.4

For an ordered set W = $\{x_1, x_2, \ldots, x_k\} \subseteq V(G)$ and a vertex v in G, the k-vector $r_G(v/W) = (d_G(v,x_1), d_G(v,x_2),\ldots\ldots d_G(v,x_k))$ is called the representation of v with respect to W. The set W is a **resolving set** for G if and only if no two distinct vertices of G have thesame representation with respect to W. The metric dimension of G, denoted by, dim(G), isthe minimum cardinality over all resolving sets of G. A resolving set of cardinality dim(G) is called a basis.

Definition 2.5

Let G be a connected graph. A set S ⊆ V(G) is a **locating set** of G if for every two distinct vertices u and v of V (G) \ S, $N_G(u) \cap S$ G $N_G(v) \cap S$. The locating number of G, denoted by ln(G), is the smallest cardinality of a locating set of G. A locating set of G of cardinality ln(G) is referred to as an ln-set of G.

Definition 2.6

A **resolving dominating set** as a set S of vertices of a connected graph G that is both resolving and dominating. The cardinality of a minimum resolving dominating set is called the resolving domination number of G and is denoted by $\gamma_R(G)$. A resolving dominating set ofcardinality $\gamma_R(G)$ is called a γ_R-set of G.

Definition 2.7

A **locating-dominating set** is a locating subset S of V(G), which is also dominating set in a connected graph G. The minimum cardinality of a locating-dominating set in G, denoted by $\gamma_L(G)$ is called the L-domination number of G. Any L-dominating set of cardinality $\gamma_L(G)$ is then referred to as a γ_L-set of G.
Let G be a connected graph. A set S ⊆ V(G) is a **strictly locating set** of G if it is a locating set of G and $N_G(u) \cap S$ G S, for all u ∈ V(G)\S. The strictly locating number of G, denoted by sln(G), is the smallest cardinality of a strictly locating set of G. A strictly locating set of G of cardinality sln(G) is referred to as sln-set of G.

Definition 2.8

A **resolving hinge total dominating set** as a set S of vertices of a connected graph G that is both resolving and hinge total dominating. The cardinality of a minimum resolving hinge total dominating set is called the resolving hinge total domination number of G and is denoted by $\gamma_{Rht}(G)$. A resolving hinge total dominating set of cardinality $\gamma_{Rht}(G)$ is called a γ_{Rht}-set of G.

Definition 2.9

A **locating hinge total dominating set** is a locating subset S of V(G), which is hinge total dominating set in a connected graph G. The minimum cardinality of a locating hinge total dominating set in G, denoted by $\gamma_{Lht}(G)$ is called the L-hinge total domination number of
G. Any L- hinge total dominating set of cardinality $\gamma_{Lht}(G)$ is then referred to as a γ_{Lht}-set ofG.

Definition 2.10

A **strictly locating hinge total dominating** is a strictly locating subset S of V (G) which is also a hinge total dominating set in a connected graph G. The minimum cardinality of a strictly locating hinge total dominating set in G, denoted by γ_{SLH}, is called the SLHT- domination number of G. Any SLHT-dominating set of cardinality $\gamma_{SLH}(G)$ is then referred toas a γ_{SLH}-set of G.

Definition 2.11

Let G be a connected graph. A set S ⊆ V(G) is **strictly resolving HTD set** of G if it isa resolving HTD set of G and $N_G(u) \cap S$ G S for u ∈ V (G)\S. The strictly resolving HTD number of G, denoted by $\gamma_{SRH}(G)$, is the smallest cardinality of a strictly resolving HTD setof G. A strictly resolving dominating set of G of cardinality $\gamma_{SRH}(G)$ is referred to as γ_{SRH}-set of G.

III. JOIN OF TWO GRAPHS USING HTD

The join of two graphs G and H, denoted by G + H is the graph such that V(G + H) = V(G) ∪ V(H) and E(G + H) = E(G) ∪ E(H) ∪ { mn : m ∈ V(G), n ∈ V(H) }.

THEOREM 3.1

Let G and H be non-trivial connected graph. A set W ⊆ V(G + H) is a resolving set of G + H if and only if $W = W_G \cup W_H$ where $W_G \subseteq V(G)$ and $W_H \subseteq V(H)$ are the locating set ofG and H respectively, where W_G or W_H is a strictly locating set.

THEOREM 3.2

Let G and H be connected graphs. Then D ⊆ V(G + H) is a hinge total dominating set in G + H if and only if at least one of the following statements is satisfied:

1. D ∩ V(G) is a dominating set in G.
2. C ∩ V(H) is a dominating set in H.
3. C ∩ V(G) G ∅ and C ∩ V(H) G ∅.

THEOREM 3.3

Let G and H be non-trivial connected graphs. A set W ⊆ V(G + H) is a resolving hinge total dominating set of G + H if and only if W is a locating hinge total dominating set of G + H.

Proof:

Let us assume that W is a resolving hinge total dominating set of G + H. Then W is aresolving set of G + H.
By Theorem 3.1, $W = W_G \cup W_H$ where $W_G \subseteq V(G)$ and $W_H \subseteq V(H)$ are locating setsof G and H, respectively, where W_G or W_H is a strictly locating set.
Since W is a hinge total dominating set of G + H, W_G and W_H are also hinge totaldominating sets of G and H, respectively.
By Theorem 3.1, W is a locating hinge total dominating set of G + H.

The converse is also true and it follows immediately from Theorem 3.1 andTheorem 3.2 iii.

THEOREM 3.4

Let G and H be non-trivial connected graphs. A set W ⊆ V(G + H) is a resolving hinge total dominating set of G + H if and only if $W = W_G \cup W_H$ where $W_G = V(G) \cap W$ and $W_H = V(H) \cap W$ are locating sets of G and H, respectively, where W_G or W_H is a strictly locating set.

Proof:

Necessary Part:

Suppose that W is a resolving hinge total dominating set of G + H. Then W is a resolving set of G + H.
By Theorem 3.1, $W_G = W \cup V(G)$ where $W_G \subseteq V(G)$ and $W_H \subseteq V(H)$ are locating sets of G and H, respectively, where W_G or W_H is a strictly locating set.
Since W is a hinge total dominating set of G + H, W_G and W_H are also hinge total dominating sets of G and H respectively.
By Theorem 3.3, W is a locating hinge total dominating set of G + H. (i.e.) $W = W_G \cup W_H$ where $W_G = V(G) \cap W$ and $W_H = V(H) \cap W$ are locating sets of G and H, respectively, where W_G or W_H is a strictly locating set.

Sufficient Part:

Let $W = W_G \cup W_H$ where $W_G = V(G) \cap W$ and $W_H = V(H) \cap W$ are locating sets of G and H, respectively, where W_G or W_H is a strictly locating set.
By theorem 3.2, W is a dominating set of G + H.Let $u,v \in V(G + H)\backslash W$ with u G v
Case 1. $u, v \in V(G)$

Since W_G is a locating set of G, $N_G(u) \cap W_G$ G $N_G(v) \cap W_G$.Hence, $r_{G+H}(u/W)$ G $r_{G+H}(v/W)$.
Case 2. $u, v \in V(H)$

Since W_H is a locating set of G, $N_H(u) \cap W_H$ G $N_H(v) \cap W_H$.Hence, $r_{G+H}(u/W)$ G $r_{G+H}(v/W)$.
Case 3. $u \in V(G)$ and $v \in V(H)$

$r_{G+H}(u/W) = (d_{G+H}(u,w_1), d_{G+H}(u,w_2)\ldots\ldots\ldots d_{G+H}(u,w_n),1,1,\ldots\ldots1)$

$r_{G+H}(v/W) = (1,1,\ldots\ldots1, d_{G+H}(v,w_1), d_{G+H}(v,w_2)\ldots\ldots\ldots d_{G+H}(v,w_n))$

Suppose there exists $j \in \{1, 2, . . . , n\}$ such that $d_{G+H}(u,w_j)$ G 1 or there existsk $\in \{1, 2, \quad , \quad m\}$ such that $d_{G+H}(v,u_k)$ G 1. Hence, $r_{G+H}(u/W)$ G $r_{G+H}(v/W)$.
Therefore, W is a resolving set of G + H. Accordingly, W is a resolving hinge totaldominating set of G + H.

IV. CONCLUSION

This study introduced us some new concepts of locating hinge total domination andresolving hinge total domination in the join of two graphs.

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